Spectrum Trading in Cognitive Radio Network: An Agent-based Model under Demand Uncertainty

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Abstract—In this paper, we propose a general model of agent-based spectrum trading, where an agent plays a third-party role in the spectrum trading process. Providing service to Secondary Users (SU) with spectrum bought from Primary Users (PU), the agent makes profit in the spectrum trading process. We address the challenge of finding the most profitable strategy of agent(s) when spectrum demand is uncertain. We first address this challenge for the secondary network where a single agent operates, and extend it to a multiple-agent system. To our best knowledge, this is the first solution to agent-based spectrum trading considering demand uncertainty.

I. INTRODUCTION

To improve the utilization of spectrum, cognitive radio is considered as the most promising technology [1]. In cognitive radio networks, Primary Users (PU) who possess licenses can share the spectrum with Secondary Users (SU) who do not.

In addition to traditional spectrum trading, an agent can bring us significant advantages. First, it does not require high intelligence of SUs, because SUs do not need to perform spectrum sensing or price negotiation with PUs. Second, agent-based model can reduce the network overhead and control information transmission between PUs and SUs [2]. Third, an agent with greater coverage can provide SUs more spectrum trading opportunities. Forth, the function of an agent can be implemented on the existing infrastructures, like BS or AP.

In this paper, we keep our model to be general so that it can be implemented in any future system with CR technology. For example, in 802.22 system, agent can be built on the base station, which leases (or senses) spectrum of VHF/UHF TV bands and serve all its associated Consumer Premise Equipments (CPEs) without any harmful interference to TV receivers (Fig. 1).

In order to focus on the analysis of agent behavior we suppose that PUs and the agent have negotiated that PUs provide spectrum to agent by an average price \( S \) for per unit spectrum (e.g., MHz). And we derive the demand function by adopting the concept of Acceptance Probability \( A(W, P) \), which reflects the willingness of secondary user to accept the service for bandwidth \( W \) and price \( P \) charged by the agent [2].

However, the randomness of the network has not been considered in the demand function. To be specific, agent may be not aware of every SU’s preference on bandwidth nor the number of SUs in each trading period (e.g., one hour, [3]). To obtain this optimal strategy, two different situations need to be analyzed: only one agent or multiple agents operates or compete in the network. So, we naturally define them as noncompeting agents and competing agents, respectively.

For brief preview, we summarized our main contributions as follows:

- An agent-based spectrum trading model considering demand uncertainty in cognitive radio network is proposed.
- In single-agent system, we provide a general solution and algorithm to achieve the optimal strategy.
- In multiple-agent system, we show that a pure strategy Nash equilibrium can be reached by a distributed algorithm.

The rest of this paper is organized as follows: In Section II, we formulate the problem and provide solutions to the single-agent system. In Section III, we formulate multiple agents’ competition by a log-supermodular game. At last, we present our simulation results in Section IV and conclude our work in Section V.

II. SPECTRUM TRADING WITH SINGLE AGENT

A. System Model

In this paper, our model is deployed in the CRN to mediate the secondary users’ satisfaction and agents’ profit. SU’s satisfaction is reflected by the acceptance probability \( A(W, P) \), which is decreasing with per unit price \( P \) but increasing with the bandwidth \( W \). The agent is expressed as \( \Pi(Q, P) \), where \( Q \) denotes spectrum leasing quantity.

1) Acceptance Probability: As indicated previously, SUs’ satisfaction is reflected by accepting the uniform service (W is constant) from an agent with a corresponding probability. From SU’s point of view, the service of the agent is acceptable only if the price and service bandwidth are reasonable. So

Fig. 1. Exemplary agent deployment in 802.22 system
we have to design an appropriate function of acceptance probability $A(W, P)$ reflecting SU’s rational behavior.

Mathematically this rationality can be formulated as:

$$\frac{\partial A(W, P)}{\partial W} \geq 0, \quad \frac{\partial A(W, P)}{\partial P} \leq 0$$

$$\lim_{W \to \infty} A(W, P) = \alpha, \quad \lim_{P \to 0} A(W, P) = 0 \quad (1)$$

$$\lim_{P \to \infty} A(W, P) = 0, \quad \lim_{P \to 0} A(W, P) = \alpha$$

Where $\alpha \leq 1$ denotes SUs’ preferences on the service. Several candidates are available and we choose the exponential expression [2]:

$$A(W, P) = ae^{-PW} \quad (2)$$

2) Demand Uncertainty: After defining the acceptance probability, we can formally derive the demand function $D(P)$ of the secondary users as ($N$ denotes the number of SUs):

$$D(P) = \sum_{i=1}^{N} A(W, P)W \quad (3)$$

As mentioned previously, the natural randomness of the secondary network is considered in our model. For example, given the service $(W, P)$, SUs may accept it with different probabilities (differed by $\alpha$). From the agent’s point of view, the spectrum demand of secondary users will be uncertain.

If the preferences of SU uniformly equal to $\alpha$ but the number of SUs $N$ is uncertain, the spectrum demand function can be expressed in a multiplicative form:

$$D(P) = d(P)e = \sum_{i=1}^{N} A_i(W, P)W = ae^{-PW} WN \quad (4)$$

Where deterministic part is $d(P) = ae^{-PW}W$ and random part is $N$. If the number of SUs $N$ keeps to be a constant, but SUs’ preferences $\alpha$ are randomly distributed in $[0,1]$, demand function can be expressed in a similar form except for the different expressions of $d(P)$ and $e$ (for details, see [8]). Here we limit our analysis only for the former case.

3) Profit of the Agent: We first suppose that all spectrum bought (leased) from PUs with amount $Q$ and per unit(e.g. MHz) price $S$ are all sold to SUs with price $P$, then the income will be $(P - S)Q$. But due to the agent’s uncertainty on its customers’ demand, spectrum the agent bought may exceed the need of SUs and this part of spectrum can be expressed as $E[(Q - D(P)\epsilon)\epsilon]$. $E[\cdot]$ denotes the mathematical expectation and $[\cdot]^+ = \max[0, \cdot]$. So we obtain the final expression of agent’s expect profit:

$$\Pi(Q, P) = (P - S)Q - PE[Q - d(P)\epsilon]^+ \quad (5)$$

In the above equation, $g(x)$ and $G(x)$ denote the pdf and cdf of the random variable $\epsilon$ respectively.

4) Spectrum Trading Process: This agent-based spectrum trading process is composed of three steps:

- **Step1:** The agent leases spectrum $Q^*$ from PUs with average price $S$.
- **Step2:** The agent broadcasts its service information $(W, P^*)$ in the service area.
- **Step3:** When SU receives the information, it accept the service in the probability $A(W, P^*)$.

B. General Solution

Mathematically the problem of finding agent’s optimal strategy can be expressed as:

$$(Q^*, P^*) = \arg \max_{(Q, P)} \Pi(Q, P) \quad (6)$$

For the analysis simplification, we transform the parameter $Q$ to $Z$ by letting $Z = Q/d(P)$. Given $(Q^*, P^*)$, optimal strategy $(Q^*, P^*)$ is equal to $(Z^*, d(P^*))$. So we equivalently rewrite our problem as:

$$(Z^*, P^*) = \arg \max_{(Z, P)} \Pi(Z, P) \quad (7)$$

Let $\theta(Z) = \int_{0}^{Z} G(x)dx$, we can also rewrite Eq.(5) as:

$$\Pi(Z, P) = (P - S)Zd(P) - Pd(P)\theta(Z) \quad (8)$$

Then the general solution of optimal strategy are obtained:

$$Z^*(P) = G^{-1}(\frac{P - S}{P}) \quad (9)$$

$$P^*(Z) = \frac{SZ}{\theta(Z) + W} + W \quad (10)$$

Optimal strategy $(Z^*, P^*)$ can be obtained at the intersection points of Eq(9) and Eq(10). First, we find that both Eq(9) and Eq(10) are increasing functions [8]. Results are naturally proved by $\theta(Z) \leq ZG(Z)$ and $0 \leq G(Z) \leq 1$. Then we find that Eq(9)’s concavity depends on the hazard rate function $r(Z)$ of $\epsilon$. In particular, for uniform, normal, logistic, chi-squared and exponential distributions, $Z^*(P)$ is all concave [4].

C. An Example of Uniform Distribution

1) Solution: In this section, we will solve the problem defined by Eq(7) for uniform distribution of $\epsilon$. As an example, we let this random variable represents the number of secondary users in different trading periods, and suppose that it is uniformly distributed on $[A, B]$, where both A and B are positive integers. With Eq(9) and Eq(10) we can find the optimal strategy of the agent:

$$P^*(Z) = \frac{2SZ(B - A)}{2ZB - Z^2 - A^2} + W, \quad A \leq Z \leq B$$

$$Z^*(P) = \frac{B - A}{P} S, \quad S + W \leq P \leq \frac{2SB}{B + A} + W \quad (11)$$

Above two equations are both concave in price $P$ [8], and we plot them in Fig.2. Obviously at least one intersection point exists and moreover it is unique.

**Theorem 1:** If the demand function of secondary users can be expressed as $D(P) = d(P)\epsilon$, where $\epsilon$ is a random
variable uniformly distributed on \([A,B]\). Then the agent has an unique optimal strategy \((Q^*,P^*) = (Z^*d(P^*),P^*)\) defined by the intersection point of Eq(9) and Eq(10).

Proof: We present proof in our technical report [8].

2) Algorithm: We have shown that there is an unique optimal solution \((Q^*,P^*) = (Z^*d(P^*),P^*)\). In practice, we provide an iterative algorithm for each agent to find this optimal strategy: starting with price \(P_0 = W + 2SB/(B + A)\). In the \(k_{th}\) iteration, we calculate \(Z_k = Z^*(P_{k-1})\) and \(P_k = P^*(Z_k)\), then the sequence \((Z_k, P_k)\) converges to \((Z^*, P^*)\) [8].

III. SPECTRUM TRADING WITH MULTIPLE AGENTS

A. System Model

In this section we consider system in which multiple agents compete for customers(SUs). We assume that \(M\) agents are providing services to \(N\) SUs, where \(M\) is constant while \(N\) is uncertain (Fig.3). Receiving a service offer \((W_i, P_i)\), SU’s willingness to accept this offer is still modeled by Acceptance Probability \(A_i(W_i, P_i)\). This concept is naturally extended from the previous section, except for the subscript \(i\) denoting different agents. Open market enables SUs to make options among different agents, but definitely complicates our problem. Indeed, the spectrum demand of agent’s customers is determined by not only its own service \((W_i, P_i)\), but also other agents’ services: \((W_j, P_j), \forall j \neq i\).

![Fig. 2. Intersection Point of \(P^*(Z)\) and \(Z^*(P)\)](image)

![Fig. 3. General Model of Multiple Agents](image)

1) Acceptance Probability: Unlike the single-agent system, we have to consider \(M\) agents this time. So we denote their prices charged and bandwidth provided by two vectors: \(\mathbb{P} = \langle P_1, P_2...P_M\rangle\) and \(\mathbb{W} = \langle W_1, W_2...W_M\rangle\). Then the probability of SU\(_j\) to accept the service from agent, can be extended from the single-agent system (see Eq(2)) as:

\[
A_j^i(\mathbb{W}, \mathbb{P}) = \alpha_j e^{-P_i/\mathbb{W}_j} \prod_{k=j+1}^{M} e^{-W_k/P_k}
\]

(12)

In this expression, \(\alpha_j \leq 1\) is used to denote the preference of SU\(_j\) to access the spectrum via an agent. We adopt this expression in multi-agent model, because the first part \(\alpha_j e^{-P_i/\mathbb{W}_j}\) satisfies the economic properties defined in Eq(1) and represents SU\(_j\)’s rationality in spectrum decision. In addition, the second part \(\prod_{k=j+1}^{M} e^{-W_k/P_k}\) represents the influences of agent,’s competitors’ presences on SU\(_j\)’s decision. Note this influence diminishes when all competing services’ price-bandwidth ratio \(\phi_k = P_k/\mathbb{W}_k\) approach infinite high, i.e., \(\prod_{k=j+1}^{M} e^{-W_k/P_k}\) approaches one and Eq(12) becomes exactly Eq(2). This acceptance probability meets economic properties:

\[
\begin{align*}
\frac{\partial A_j^i(\mathbb{W}, \mathbb{P})}{\partial \phi_i} & \leq 0 \\
\frac{\partial A_j^i(\mathbb{W}, \mathbb{P})}{\partial \phi_k} & \geq 0
\end{align*}
\]

(13)

2) Spectrum Demand: After defining secondary users’ willingness to accept the service, we can derive the spectrum demand function of \(N\) secondary users for agent, as:

\[
D_i(\mathbb{P}) = \sum_{j}^{N} A_j^i(\mathbb{P})W_j
\]

(14)

For the sake of simplicity, we consider that secondary users have uniform preferences, i.e., \(\alpha_j = \alpha\) and \(A_j^i(\cdot) = A_i(\cdot)\) for all SU\(_j\). The spectrum demand can be simplified as:

\[
D_i(\mathbb{P}) = A_i(\mathbb{P})W_iN = d_i(\mathbb{P})N
\]

(15)

3) Profit of the Agent: In this multiple agents system, we still consider the system where the number of SUs \(N\) is uncertain. Similarly the strategies of each agent still consists of two elements: the price \(P\) and spectrum quantity \(Q\). Note that agent,’s profit is only influenced by its competitors’ strategy of price but not the strategy of spectrum quantity. So the formal definition of the agent,’s profit can be expressed as:

\[
\Pi_i(Q_i, \mathbb{P}) = (P_i - S_i)Q_i - P_iE[Q_i - D_i(\mathbb{P})]^+
\]

(16)

4) Nash Equilibrium of Agent Strategies: We address the problem of finding Nash equilibrium of strategies given the distribution of \(N\). Since the expected profit of agent, does not depend on its competitors’ decisions on \(Q\), we let \(\Pi_i(Q_i, \mathbb{P})’\)’s partial derivative on \(Q\) equal to 0 and get the optimal \(Q\) under a given price vector as:

\[
Q_i^* = d_i(\mathbb{P})G^{-1}(R_i)
\]

(17)

where \(R_i = (P_i - S_i)/P_i\). The above equation tells us a fact that given an optimal price vector \(\mathbb{P}^*\), we can obtain the optimal
bandwidth quantity $Q_i$ agent, should buy from PUs. Thus, we can reduce the original problem of finding $(Q_i, P_i^*)$ to a reduced problem of finding $P_i^*$. So the Nash equilibrium of strategies of this game where agents competing for secondary users can be defined as finding price vector $P^*$ such that for each agent $i$:

$$\Pi_i(P^*) = \Pi_i(P_i^*, P_{-i}^*) \geq \Pi_i(P_i, P_{-i})$$

(18)

where $P_{-i}$ represents the price vector of all agent$_i$’s competitors. Indeed, substituting equation (17) to equation (16), we can simplify the expression of expect profit:

$$\Pi_i(P) = (P_i - S_i)d_i(P)G_i^{-1}(R_i) - P_iE[d_i(P)G_i^{-1}(R_i) - d_i(P)]$$

(19)

In above, the first term: $\Pi_i^{dr}(P) = (P_i - S_i)d_i(P)$, which is independent of random $N$ and represents the deterministic part of expect profit. The second term: $L_i^{nd}(R_i) = \frac{1}{R_i} \int_0^{R_i} xg(x)dx \leq E[N]$ represents the random part of expect profit due to the uncertainty of $N$.

B. Existence of Nash Equilibrium

In order to find a Nash equilibrium of competing agents’ strategies (see Eq.18), we first introduce the concept of Supermodular Game [5][6], which is better known as games with strategic complementarities. To be specific, when we try to optimize multiple endogenous variable (prices charged by agents), then all of them are complements if their increases are mutually reinforcing.

*Increasing difference*: Function $f(x, t) : X \times T \rightarrow \mathbb{R}$ has increasing difference in $(x,t)$, if $f(x', t') - f(x, t) \geq f(x', t) - f(x, t)$ or $\frac{df(x, t)}{dx} \geq 0$, for all $x' \geq x$ and $t' \geq t$.

*Supermodular Game*: Each player $i$ and its competitors’ strategy set $P_i$ and $P_{-i} \in \mathbb{R}$, and payoff $\Pi_i(P_i, P_{-i})$ has increasing difference in $(P_i, P_{-i})$, i.e., incremental payoff to choose a higher strategy is increasing with competitors’ strategies.

In fact, our reduced game is not supermodular but we can prove that it is log-supermodular, that is, the log payoff is supermodular [6]. Although log-supermodularity is essentially weaker than supermodularity, it’s enough to ensure the existence of Nash equilibrium in our reduced game. Before presenting the theorem, we first introduce two conditions:

**Condition A**: For each $i=1, ..., M$, the function $\log d_i(P)$ has increasing differences in $(P_i, P_k)$ for all $k \neq i$.

**Condition B**: Each agent chooses its price $P_i$ from a closed interval $[P_{min}, P_{max}]$.

Given conditions A and B, we can present the following theorem ensuring the existence of Nash equilibrium:

**Theorem 2**: If Conditions A and B are satisfied, then we have following results:

(a) Nash equilibrium $P^*$ for the reduced game exists, and its corresponding strategy $(P^*, Q(P^*))$ is a Nash equilibrium for the original game.

(b) If multiple Nash equilibria exist, there is a smallest and largest equilibrium $P_\text{Min}$ and $P_\text{Max}$, respectively.

(c) The equilibrium $P^*$ is preferred by all agents.

**Proof**: We present proof in our technical report [8].

C. Algorithm

As we proved, the price strategies of competing agents have log-supermodular properties so that $P^*$ can be computed easily by a well-known scheme [7]: Starting with an arbitrary price vector $P^0 \geq P^\text{Max}$, e.g., $P^0 = P^{\text{Max}} = (P^{\text{Max}}_1, ..., P^{\text{Max}}_M)$. In the $k_{th}$ iteration $P^k$ is obtained from $P^{k-1}$ by $P^k = \arg \max_{P_i} \Pi_i(P, P_{-i}^{k-1})$, then the sequence $(P^k, k > 0)$ converges to $P^*$. Based on this log-supermodular game model, fortunately we can avoid multiple-equilibrium case by setting the initial price vector to $P^{\text{Max}}$, and it converges to the greatest Nash equilibrium $P^*$ which is preferred by all agents.

IV. SIMULATION RESULTS

In this section, we use network simulations to evaluate the performance of our algorithm, and study the impact of demand uncertainty. First we implement the iterative algorithm to find the optimal strategy of a single agent. Simulation parameters are set as: $S = 10 \$/MHz, $W = 10$ MHz, $\delta = 0.01$, and $N$ is uniformly distributed in $[10, 90]$. And we assume secondary users’ have uniform preference $a = 1$.

We simulate our algorithm to find the optimal strategy $(Q^*, P^*)$ of the agent, and show its converging path with expect profit contour in Fig. 7. The result of this simulation shows that optimal price $P^* = 23.029 \$/MHz and optimal quantity of idle spectrum agent should lease $Q^* = 55.245$ MHz bandwidth. The expected profit of this strategy $\Pi(Q^*, P^*) = 425.010$ $\$ and iteration time $k = 8$.

Although we have shown that this algorithm can achieve optimal expected profit, the realized profit is usually less than the maximum achievable profit due to the uncertainty of SU’s spectrum demand. According to microeconomic theory, the maximum achievable profit can be achieved if agent can exactly predict the number of SUs in each trading period. We define the ratio of realized profit over maximum achievable profit as optimality and the gap between them as optimality gap.

We quantify the uncertainty by the variance of the uniform distribution $g(x)$. So we can study its impact by simulation. For the fairness, we fix the mean value of $N$ as $N_{AVR} = 500$ and simulate with variance $\sigma$ from 1 to 1000, and each point is averaged by $10^6$ rounds.

Fig. 4 shows impact of demand uncertainty on the strategies’ optimality and realized profit, and we find out three interesting results:

- The optimality and realized profit drop with the uncertainty of spectrum demand (variance of $\sigma$).
- Even in the most uncertain case (variance equals $10^3$), the optimality is still above 89%.
- The optimality and realized profit saturate when the variance $\sigma < 10$.

In order to study the impact of this uncertainty further, we present the optimal strategies as a function of variance $\sigma$ in Fig. 5 and a function of bandwidth $W$ in Fig.8. We find an interesting result: the optimal price of spectrum increases when the number of SUs is more unpredictable, while the optimal
quantity of spectrum decreases. Intuitively it shows that when the market becomes unpredictable the agent prefers higher profit of per unit product (bandwidth) to higher sales.

In Fig. 6, we present the realized profit of the agent as a function of the average number of secondary users \((N_{AVR})\). We plot three curves with \(\alpha=1\), \(\alpha\) uniformly selected from \([0.7,1.0]\) and \([0.5,1.0]\), respectively. As we expected, the profit agent makes increases linearly with the mean value of \(N\). We can also see that the randomness of \(\alpha\) again decreases the profit of the agent.

To study the multiple-agent case, we simulate the spectrum trading process with two agents, i.e., \(M=2\). We suppose that the number of secondary users is uniformly distributed in \([100, 200]\), and their preferences are all set to be unit. Other parameters can be found in our technical report [8].

Results shown in Fig. 9 indicate that strategies of two agents converge to different sets. In equilibrium, agent\(_1\) provide service with lower price but it leases more spectrum from PUs. Because its lower price attract more secondary users to select service from it: in equilibrium the acceptance probability of secondary users for agent\(_1\) is 11.38\% while that for agent\(_2\) is only 9.84\%, which are shown in Fig. 10.

In order to study the impact of competition, we compare the realized profits of two agents with competition and without competition. The number of secondary users is still uniformly generated in \([100, 200]\), and their preferences are all set to be unit. We plot the average profits of two agents in Fig. 11, which shows competition greatly decreases profits of agents.

V. CONCLUSION AND FUTURE WORK

In this paper, we propose an-agent based model for spectrum trading in cognitive radio network. The optimal strategy of agent is obtained under the circumstance where the spectrum demand of secondary users is uncertain.

For single-agent system, our solution is proved optimal and demand uncertainty decreases agent’s profit. It’s interesting that when the market becomes more unpredictable the agent prefers higher profit of per unit product (bandwidth \(W\)) to higher sales. When multiple agents operate, we can obtain the equilibrium of agents’ strategies. However, profits made by two competing agents are reduced comparing the profits made when they are operating independently.

We try to derive the optimal solutions for agents in an uncertain market. We give general solutions, but only uniform distribution is deeply analyzed. For other distributions (e.g., normal distribution), more analysis is needed in future work. In addition, the security issues of agent-based spectrum trading will also be included.

REFERENCES